**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *m* = 45 minutes and *s* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B. 0.2676 …[cdf = stats.norm.cdf(50,45,8)

pb = 1 - cdf

O/P = 0.2659855290487005]

So here for the car to be ready within the specified time,1 hr – 10 min = 60 –10 = 50 min.

Hence we can find the probability of P>= 50 for car to be ready in 1 hr.

Car not ready = 1 – p>=50

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *m* = 38 and Standard deviation *s* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans : False, as the 44 and above age group lies in 2nd (*m+2s* ) and 3rd (*m+3s* ) standard deviation of normal distribution the total number of values will be around 16% which is lower than number present in group of 38 and 44 lying in 1st standard deviation which is around 34.1%.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True [ cdf\_2 = stats.norm.cdf(30,38,6)

cdf\_2 \* 400

O/P = 36.4 =36]

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

As X1 and X2 are iid so we can,

Therefore 2(X1) = (2 μ, 4σ2) ….. [As mean is multiplied by 2 we get 2 μ this, for variance when we substitute new mean will be 2 μ which when squared will be 4]

X1 + X2 = (μ + μ, σ2 + σ2) = (2 μ, 2σ2)

2X1 - (X1+X2) = (2 μ, 4σ2) - (2 μ, 2σ2)

2X1 - (X1+X2) = (0 μ, 6σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D.

RV = 0.99

So, the probability for a and b = 1 – 0.99 = 0.01

Probability for a = 0.01/2 = 0.005

Probability for b = 0.01/2 = 0.005

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee range in millions containing 95% probability for the annual profit of the company (99.00810347848784, 980.9918965215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit (in Rupees) for the company = 171 million.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of loss from div 1 = 0.0477903522728147, Probility of loss from div 2 = 0.040059156863817086

Division 1 has higher probability of making a loss.